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Stiffness of human hair fibers

G. V. SCOTT and C. R. ROBBINS *Colgate-Palmolive Research Center, 909 River Road, Piscataway, NJ 08854.*

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Synopsis

The STIFFNESS of COMPONENT FIBERS is known to be important to the behavior of a fiber mass, but measurements are lacking in the cosmetic literature probably because of experimental difficulties with published methods. Recognizing this, we devised a simple method to compare fibers for stiffness.

A fiber with a small weight on each end is draped over a wire and the distance ("D") between the vertical legs is measured. Fibers with a wide range of thicknesses clearly showed that values of "D" relate linearly to cross-sectional areas, as expected of "stiffness." This prompted a theoretical study which yielded equations in terms of "D" for calculating, e.g., elastic bending moduli and shapes of hanging fibers.

Empirical and theoretical guides are given for selection of wire diameter and fiber weights. The average elastic modulus for bending fibers, assumed round in cross section, is approximately equal to that for stretching the same fibers. Fiber stiffness is affected by humidity and chemical treatments but is relatively unaffected by shampoos.

INTRODUCTION

The stiffness or resistance to bending of individual fibers unquestionably plays an important role in determining the behavior of any assembly of fibers. Textile literature (1–7) recognizes this importance in attempts to relate fiber stiffness to such fabric properties as flexibility, drape, handle, crease resistance and wear. Although changes in fiber stiffness must likewise affect manageability, body, combing, wave retention and handle of human hair (8, 9), the few hair measurements reported are mainly found in wool research literature (4, 10, 11). Several reasons may account for the lack of hair research in this area.

Measurement of fiber stiffness has been an experimentally difficult task. Our need for measurements arose during an investigation of effects on hair produced by polymerization within the fibers. Appreciable tensile increases were achieved, but we wished to directly measure changes in fiber stiffness. Sophisticated methods, each with appropriate theory, were reviewed in the textile literature. Many articles depended on deflection of very short fiber segments treated as cantilever beams, either end-loaded (3–5, 7, 12, 13) or center-loaded (2, 6). We tried the end-loading approach with only partial success. A "loop deformation" method (1, 6, 12) was rejected because fiber rings had to be very carefully made round, in one plane and small, 1 to 2 cm in diameter. Both beam and loop methods required precise measurement of very small deflections and forces. A dynamic procedure using short lengths of fibers as vibrating reeds (3, 6, 10, 11, 14) seemed experimentally more attractive, but we preferred a static or quasi-static (10) method for relating to hair behavior. No other methods reviewed offered better prospects for routine screening of hair treatment effects.

Changes in fiber strength are conveniently monitored by conventional tensile measurements which presumably serve as indirect measures of fiber stiffness. Controversy exists, however, as to the equivalence of elastic moduli calculated from stretching and from bending keratin fibers (3–5, 11). Unlike stretching, fiber bending involves both extension and compression with greatest strains near peripheral points of the fiber cross section. As a consequence, if fiber strength is affected chiefly in outer portions of a fiber, treatment effects may be detected more readily by stiffness than by tensile measurements.

Aside from means for estimating stiffness, a question remains as to the extent to which hair fiber stiffness can be altered by practicable hair treatments. Few current hair products of the nondamaging variety can be expected to produce more than superficial effects on stiffness. With a convenient measuring means available, however, perhaps this can be changed.

Essential working details of a simple method for measuring hair fiber stiffness were first disclosed in a very brief communication (15). The present paper describes the complete method giving information which includes a theoretical basis for equations, experimental data obtained on hair fibers and how these data relate to other measured properties of the same fibers. For easier reading, theoretical equations are derived in the Appendix with appropriate equations brought forward where needed in the text. The Appendix also contains a glossary of symbols used in this paper.

EXPERIMENTAL MATERIALS AND METHODS

Hair fibers used were from a 15-year-old Caucasian female (H), a 12-year-old Caucasian female (L) and from purchased South Korean hair (A. Klugman Inc., New York, New York). Unless otherwise specified, the fibers were equilibrated and measured in a room maintained at 60% RH, 75°F.

STIFFNESS DETERMINATION (D)

The procedure used for determining stiffness is as follows: weights are attached to each end of a fiber by threading the end through a short length of plastic tubing and inserting a tapered metal pin in the tubing to wedge the fiber. The weights of pin plus tubing on each fiber end are equal and known exactly.

The fiber is carefully draped over a wire hook and a separately hung guide bar is brought into light contact with the fiber legs to hold the fiber plane perpendicular to the optical axis of a horizontal cathetometer. The distance between the two vertical legs is measured several times by moving the distal end upwards, sliding the fiber to different contact points on the hook. The average distance in centimeters is expressed as the stiffness index (D) for that fiber.

For most human hair fibers, a recommended weight of pin and tubing is 0.1 g with a hook wire diameter of 0.75 mm.

LINEAR DENSITY DETERMINATION (L)

The fiber is measured for length to the nearest millimeter and is weighed to the nearest 0.01 mg using a Roller Smith® 3-mg Precision Balance (Federal Pacific Electric Co., Newark, New Jersey). Results are conveniently expressed as micrograms per cm (L) of fiber.

TENSILE DETERMINATION (H)

A fiber of 5-cm gauge length is extended at a rate of 0.1 in./min using an Instron® Model TM with Tension Cell A set at 10 g full scale. From the linear portion of the charted trace, the Hookean or elastic slope is estimated as g per mm extension (H) of the 5-cm fiber.

RESULTS AND DISCUSSION

Although fiber stiffness is important for overall hair performance, no convenient method for measurement of single fibers appeared to be available. Several approaches were tried in an effort to develop an empirical procedure which might be applied for evaluating hair treatment effects. Instron measurement of the work required to draw large hair loops taut between pegs was encouraging but suggested the simpler method described in this paper.

A fiber, weighted on each end, is draped over a fine wire and the distance (D) between the vertical legs is measured. The test proved useful on an empirical basis and became more acceptable with development of theory, outlined in the Appendix. The test is referred to below as the "Balanced Fiber" method and the distance between legs as the "Stiffness Index."

FIBER SHAPE

A fiber was hung in the usual way and photographed. Before theory was developed, attempts were made to shape-fit enlargements with simple curves that might empirically characterize the hanging fiber. Our lack of success here is readily explained by the complexity of the theoretical equation for fiber shape (Appendix, eq 5).

The equation was tested by measuring the enlargement for a D value and calculating values of y at assigned x values. The points coincide with the fiber shape as shown in Figure 1, confirming the theoretical treatment.

In another photo, not shown, a pronounced short-length bend just above the vertical part of the fiber had apparently no effect on the otherwise smooth inverted-U shape. A technique of comparing theoretical and photographed fiber shapes may be useful for examining individual fibers for eccentricities. For this purpose, calculations have been simplified by providing computed factors (Appendix—Fiber Shape). The distance from the hook to any point along the fiber may also be calculated by applying similar computations to eq 6, Appendix.

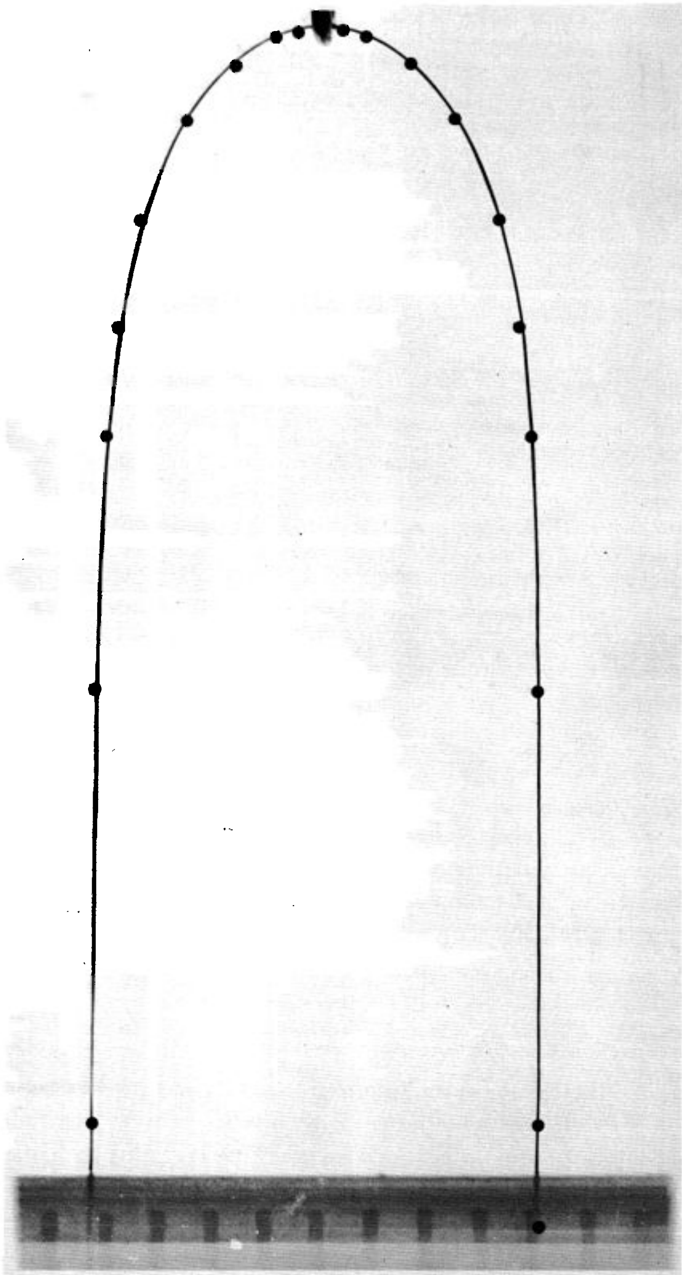

Figure 1. Photograph of a balanced fiber with calculated points superimposed

The closest analogy to the Balanced Fiber method appears to be a method developed by D. Sinclair (16). A long glass fiber is twisted slightly to form a loop which is gradually drawn tight by moving one free end of the fiber. Force is measured as a function of the distance the end is moved.

The procedure may not be suitable for hair fibers, but the same force and reaction considerations apply for a segment of the looped fiber as apply for our balanced fiber. In confirmation, we succeeded in transforming eq 5 in the Appendix into an equation identical with that derived for the looped fiber.

RELATION TO LINEAR DENSITY (L)

In developing a stiffness test a test of accuracy is required since satisfactory materials or methods are not available for reference. Fibers of larger cross-sectional area (A) were therefore assumed to be stiffer. This assumption has deficiencies, however, since fibers with the same area values may differ in stiffness because of shape or composition. For example, flat fibers bend more easily than cylindrical fibers. Nevertheless, this assumption provided a good, first test of accuracy.

Actual measurement of fiber diameters is difficult and linear densities, determined for each fiber, were considered to be proportional to cross-sectional areas of unaltered fibers. Area values may be estimated by dividing these measurements, expressed as g/cm, by 1.31 g/cm^3, the bulk density found for wool fibers (17, 18) and assumed (11, 19) appropriate for hair.

The data plotted in Figure 2 represent 24 fibers from three sources measured at 60% RH, 75°F. The only basis for selection of fibers from two individuals and Korean hair was to cover a broad range of linear densities.

Each point represents an average of four stiffness values taken at random along the fiber length. A Bartlett test (20) indicates that standard deviations for the point-to-point measurements on each of the fibers are homogeneous with a 3.7% pooled standard deviation.

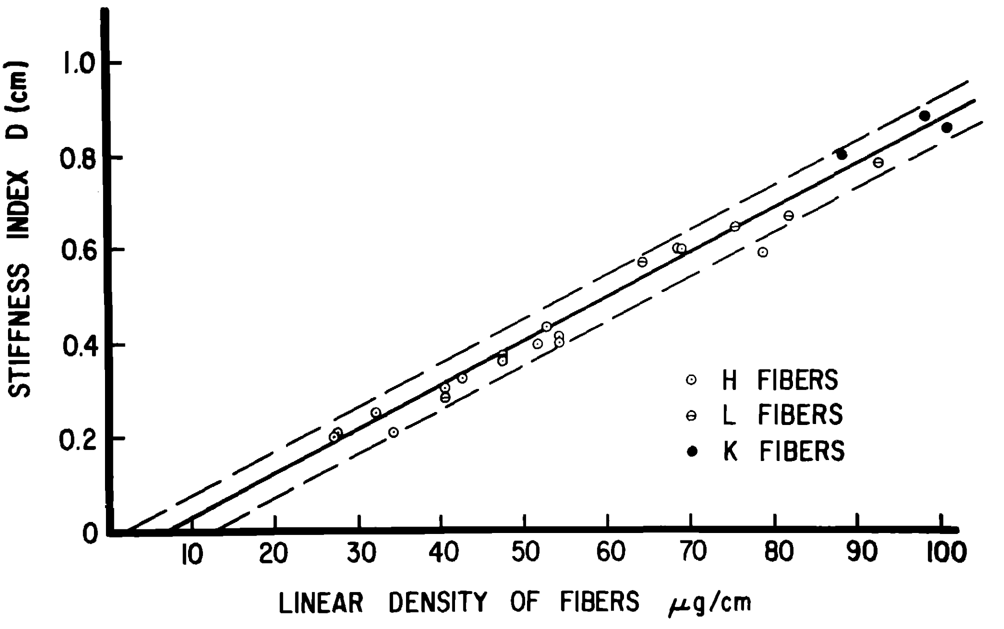

Figure 2. Effect of linear density on the stiffness index

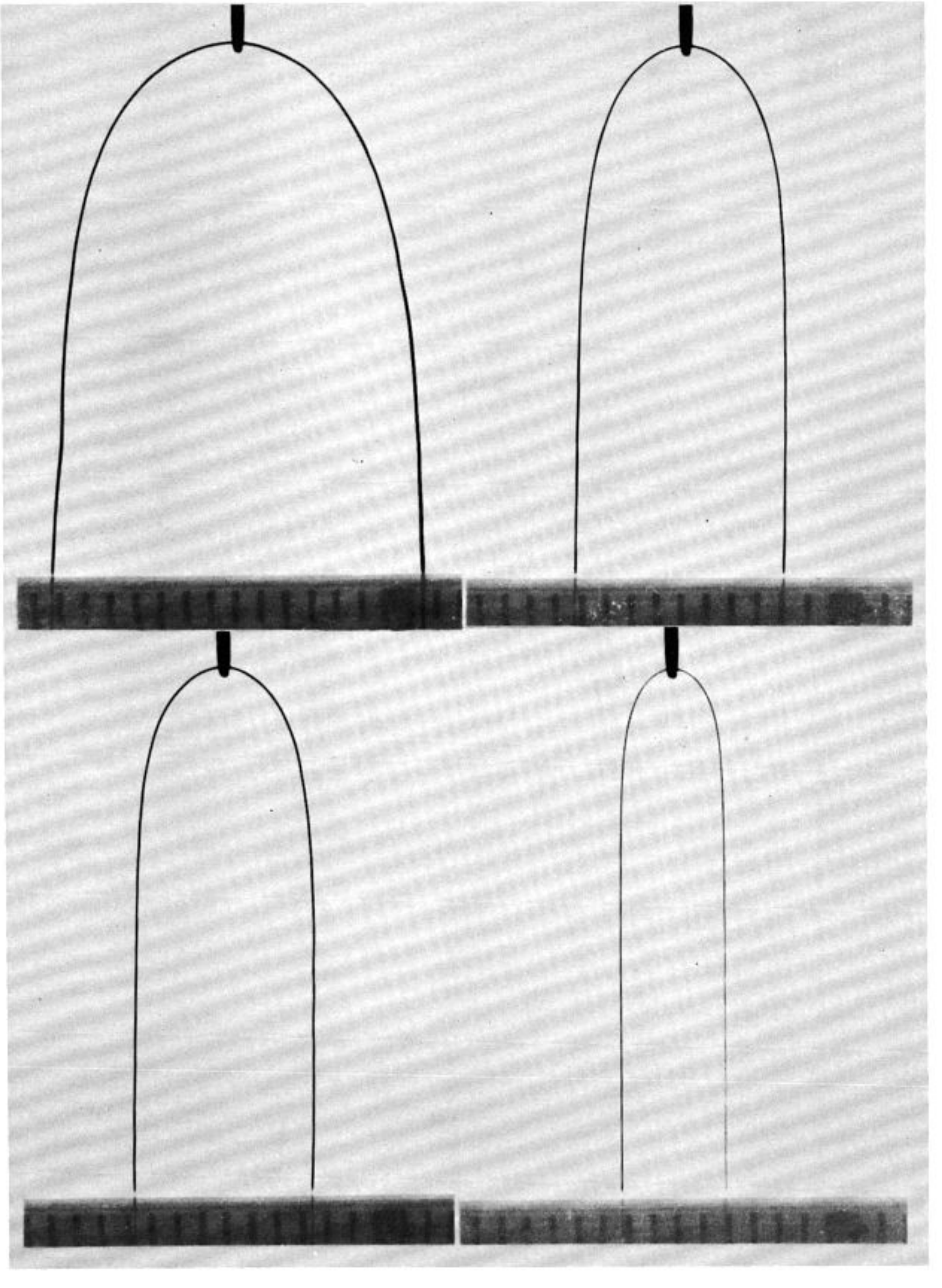

Figure 3. Photographs of balanced fibers having linear densities of 118, 71, 55 and 32 μg/cm

The straight-line relationship in Figure 2 of stiffness index to linear density and hence to approximate cross-sectional area is anticipated from eq 7 in the Appendix since the bending modulus should be nearly a constant. The scatter of points within the 95% confidence limits in Figure 2 is attributed mainly to shape differences among fibers with flatter fibers giving low and rounder fibers high stiffness values. Slight inelastic yielding of thinner fibers during measurement may account for the intercept not passing through the graph origin in Figure 2.

To more graphically illustrate the variation in stiffness from fine to coarse fibers, the photographs in Figure 3 were taken of four fibers weighted equally and with the proximal or root ends at the left of the hook. The linear densities correspond approximately to fiber diameters of 107, 83, 73 and 55 μ. It is apparent that the weight must be increased for the thickest fiber if the stiffness index D is to be measured.

EFFECT OF ATTACHED WEIGHTS

Stiffness indices were determined for three fibers with different weights attached to the fiber ends. In accordance with eq 7 and 9 in the Appendix, the stiffness index squared is plotted against the reciprocal of the attached weight for each fiber. Results in Figure 4 show linearity in agreement with theoretical prediction.

When fibers such as those shown in Figure 3 are not all measurable at the same weight, stiffness results are more conveniently compared in terms of stiffness coefficients (see

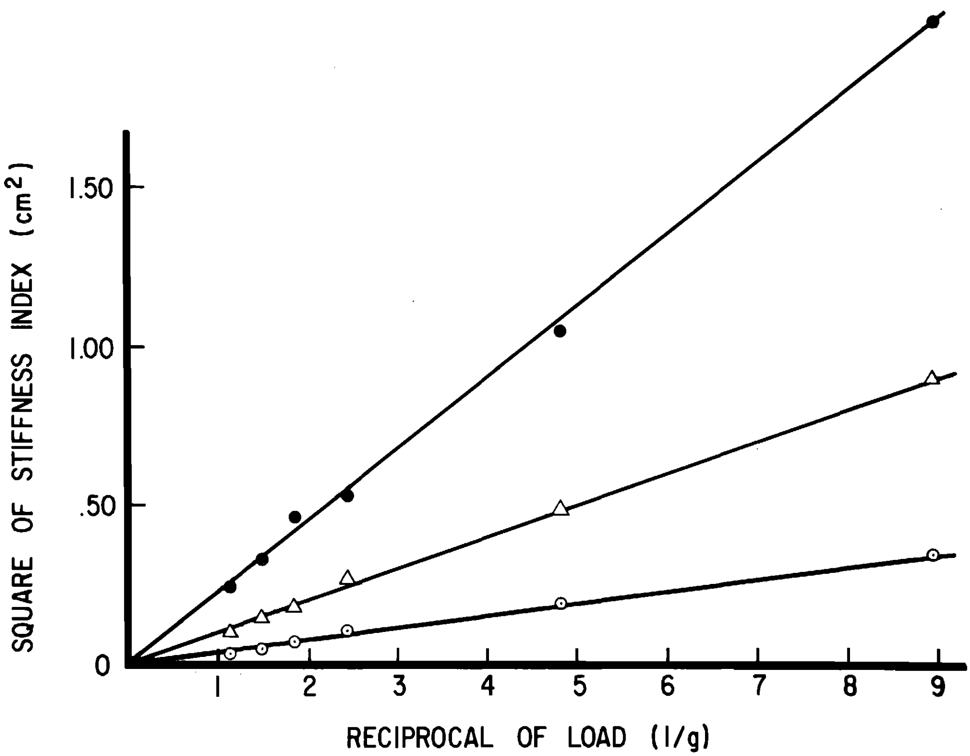

Figure 4. Effect of load on the stiffness index using functions indicated by eq 3

Appendix eq 9), which compensate for different weight loadings. Weights should be large enough to produce vertical hang without imparting a set to the fiber. As a precaution, fibers should be examined after measurement for signs of imparted set.

The effect of weight change on the stiffness index and on bending moments at the hook are derived in the Appendix. Briefly, increasing weights fourfold will double the bending moment and halve the distance between fiber legs.

EFFECT OF HOOK WIRE DIAMETER (W)

The diameter of wire used to suspend fibers was varied from 0.19 to 3.16 mm to test effects on the "D" measurement. Results in Table I show that D values are independent of wire size for most of the range covered. Careful handling of fibers is especially important with the fine wire sizes. At the largest wire size, the D measurement increases for all but the stiffest fiber.

Empirically we learned that wire size is too large if the fiber tends to orient on the support wire in a plane perpendicular to the wire. On the other hand, if a fiber tends to swivel freely, the support wire has an acceptable diameter.

An explanation of this behavior involves the problem of point and arc contacts between fiber and substrate, which is of considerable interest also for frictional studies (21, 22) using capstan methods.

With fine wires, curvature of the wire surface is greater than that of the bent fiber and theoretically contact exists only at a "point." As wire size becomes larger, curvature of the wire surface will, at some stage, equal the curvature of the bent fiber. Beyond this stage, contact between fiber and wire becomes an arc which increases in length as wire size is further increased.

Equal curvature of fiber and wire is achieved when the wire diameter (W) is equal to half the D measurement (see Appendix). "Point" contact thus exists when the ratio D/W is above two and arc contact below two.

The data in Table I provide an interesting empirical test of the D/W criterion. For wire sizes to 1.62 mm, D/W is greater than two and D values do not depend on wire size. For the 3.16-mm wire and all fibers except the stiffest, the D/W ratio is less than two and D measurements change. For the stiffest fiber, D/W = 2.18 (point contact) and the D measurement is unaffected by this wire size.

Table I
Hook Wire Diameter

Wire (cm)	0.019	0.024	0.031	0.038	0.076	0.128	0.162	0.316
Lin. Dens.				"D" Distance				
46.9 μg/cm	0.35	0.35	0.34	0.34	0.33	0.33	0.33	0.45
51.1	0.39	—	—	—	—	—	0.39	0.47
56.7	0.41	0.41	0.41	0.40	0.41	0.41	0.39	0.48
58.0	0.44	—	—	—	—	—	0.44	0.51
63.6	0.48	—	—	—	—	—	0.46	0.52
69.0	0.50	—	—	—	—	—	0.51	0.54
89.8	0.61	—	—	—	—	—	0.60	0.62
89.3	0.68	—	—	—	—	—	0.69	0.69

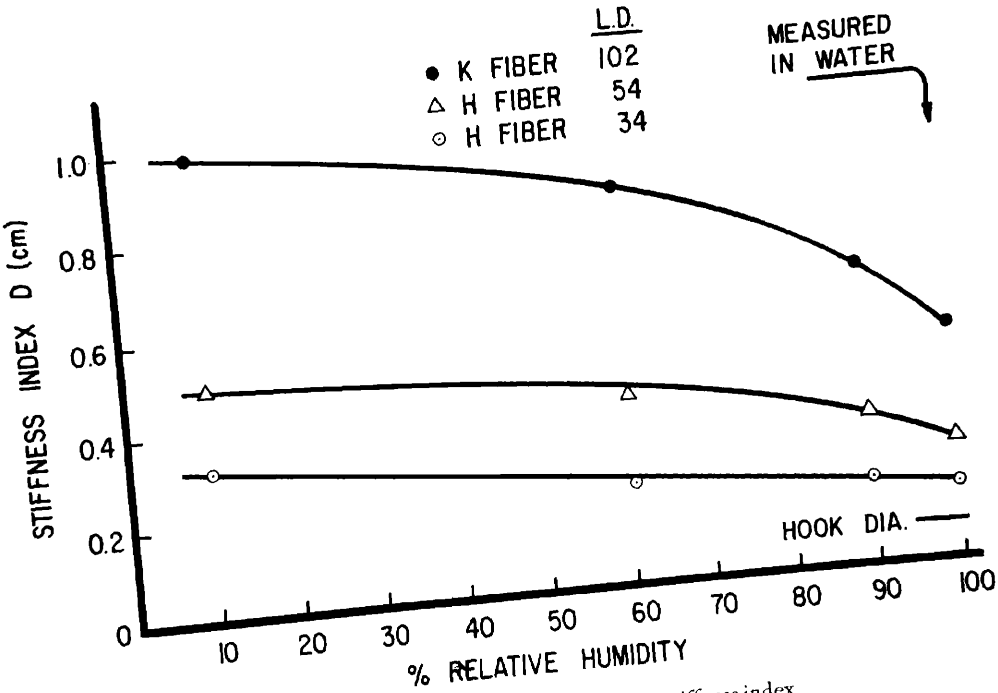

Figure 5. Effect of relative humidity on the stiffness index

EFFECT OF RELATIVE HUMIDITY

The influence of humidity is shown in Figure 5 for three fibers of different thicknesses. At the highest humidity, these fibers were measured without experimental difficulty while immersed in water. The thickest fiber showed greatest sensitivity to humidity changes while the thinnest fiber is limited in the amount of change since the "stiffness index" value in water is less than twice the wire diameter. Immersion testing requires additional study to select optimum conditions. Smaller weights can be used since wet fibers bend more easily and the fiber mass is diminished by buoyancy.

RELATION TO ELASTIC EXTENSION

An objective here was to test the strength of unaltered fibers by bending and by stretching to see how well one measurement predicts the other.

Fourteen fibers were measured for linear density and stiffness index and were then stretched with Instron equipment under conditions suitable for determining the linear or Hookean slope portion of the extension curve. When plotted, the Hookean slope data showed a good, straight-line relationship to linear densities. As expected, strength increased with thickness of fibers.

In Figure 6, stiffness indices and Hookean slopes for the 14 fibers show a satisfactory linear relationship. The prediction equation is approximately $D = 0.0151\ H$ for the dimensions used. Thus, tensile measurements on unaltered fibers may be used, to an extent, to estimate bending strengths.

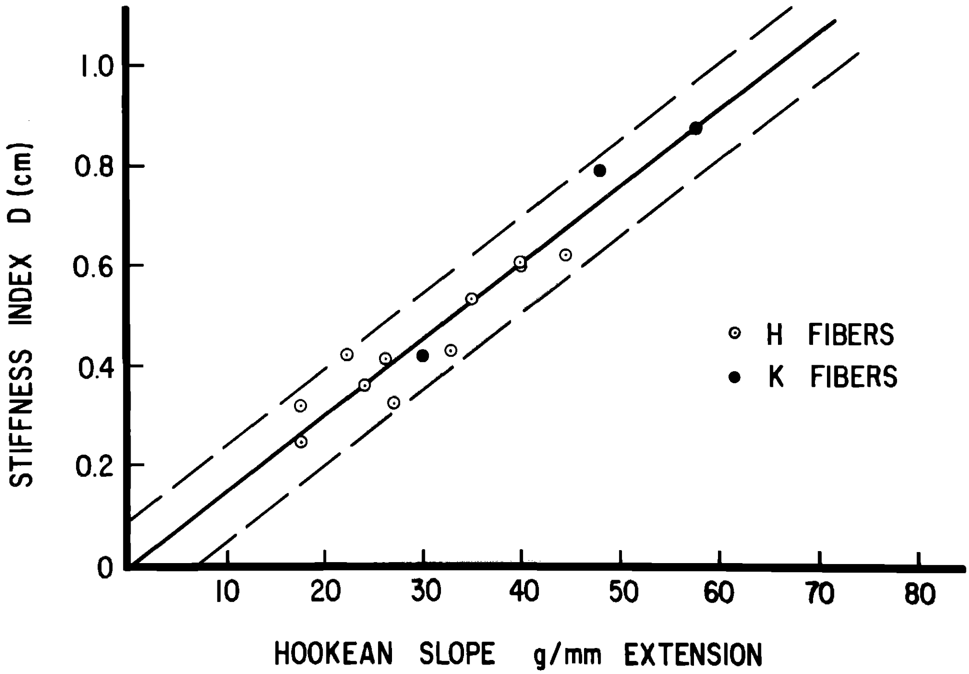

Figure 6. Relation of the stiffness index to the Hookean slope

ELASTIC MODULI

During bending, one side of a fiber is extended and the other side is compressed. For homogeneous elastic fibers with perfectly round cross sections, bending and tensile moduli would be identical and predict each other. For natural fibers such as wool or hair, however, the degree to which this identity holds is a matter of controversy (2–6, 11, 13). This is not surprising since hair fibers are oil-containing, viscoelastic, anisotropic materials which are nonuniform in cross-sectional shape and of variable thickness along their length.

Results are shown in Table II for fibers, measured at 60% RH, 75°F, and arranged according to linear densities (approximate thicknesses). The elastic moduli for bending (E_B) are calculated from eq 7 and those for stretching (E_S) from eq 8. For both calculations, the fibers are assumed to be round in cross section.

The larger spread of values for bending moduli is ascribed to greater dependence on shape factors. It is interesting that the averages for E_B and E_S are approximately equal even though the E_B/E_S ratio varies widely for the individual fibers.

The modulus values in Table II are calculated by assuming all fiber cross sections are circular. Correction for shape would increase E_B values since, in the Balanced Fiber method, bending occurs preferentially across the flattest cross section. Higher E_B values may therefore more closely represent circular fibers and truer E_B values. Accordingly, the data favors an hypothesis (4, 6) that the bending modulus is greater than the stretching modulus. The logic is that outer layers of a fiber are stiffer and play a greater role in bending than in stretching.

Table II
Elastic Moduli[a]

Fiber	Lin. Dens. $\mu g/cm$	$E_B \cdot 10^{-10}$	$E_S \cdot 10^{-10}$	E_B/E_S
K	99.5	4.23	3.68	1.15
L	94.9	3.54	3.82	0.93
K	89.2	4.29	3.43	1.25
L	71.8	4.25	3.83	1.11
H	69.2	4.11	3.75	1.10
L	67.7	3.35	3.96	0.85
L	54.6	3.60	4.12	0.88
L	52.9	4.69	3.98	1.18
H	52.6	3.74	4.03	0.93
H	42.3	3.23	4.21	0.77
L	34.4	2.89	4.33	0.67
H	31.3	3.58	3.59	1.00
Aver.	63.4	3.79	3.89	0.97
% SD	—	13.9	6.7	—

[a]Expressed as dynes/cm^2.

In stiffness studies of various natural and synthetic fibers, textile researchers occasionally include human hair fibers. Results reported for hair are shown in Table III for comparison with balanced fiber results.

Although test fibers were carefully selected and prepared, fiber-to-fiber variation in E_B for the other methods is appreciably greater than for the Balanced Fiber method. The average E_S values show much better agreement in Table III than the E_B values.

With wool fibers, the Balanced Fiber method may generally not be applicable because of insufficient fiber length. A Vibrating Reed Method (11) gave a low, fiber-to-fiber variation (12% S.D.) for wool but the E_B value of 8×10^{10} appears relatively high, presumably because of frequencies used. E_B/E_S ratios reported for wool (3–5, 11) vary from 0.4 to 3.4, possibly because of differences and difficulties in the methods.

APPLICATIONS OF THE METHOD

Although additional study is suggested, a few experiments involving dry stiffness measurements are briefly indicated below to illustrate usefulness of the method.

Table III
Elastic Moduli

Ref.	$E_B \cdot 10^{-10}$	% S.D.	$E_S \cdot 10^{-10}$	% S.D.	E_B/E_S
(4)	1.95[a]	40.9	3.57	16.8	0.55
(11)	5.35[b]	22.4	3.68	7.7	1.45
(10)	4.9[b]	—	—	—	—
(19)	—	—	3.60	—	—
S&R	3.79[c]	13.9	3.89	6.7	0.97

[a]Cantilever Beam Method.
[b]Vibrating Reed Method.
[c]Balanced Fiber Method.

A well known product with hair conditioning claims was applied to fibers in a variety of ways. A decrease in fiber stiffness resulted but the original values were restored following use of a commercially available shampoo (23).

Permanent waving of hair with a personal use product caused fiber stiffness to progressively decrease as time allowed for the reduction step was increased (23). Polymerization within fibers is also generally accomplished with an initial reduction step which weakens fibers. Nevertheless, overall increases in stiffness are achieved by proper selection of monomers and reaction conditions (23).

Proximal and distal halves of four long fibers from each of three individuals were compared for stiffness in an attempt to detect normal wear and aging effects. Unexpectedly, *distal* sections of nine fibers were stiffer and stiffness averaged 2% higher for distal halves of all fibers. Moreover, linear density was greater for distal halves of six fibers.

Bleached and untreated fibers from the same individual could not be distinguished when stiffness and linear density results were graphed. Measurement of *same* fibers before and after bleaching should be more discriminating.

Dry fiber stiffness is mainly discussed in the present paper because of its important influence on a person's hair behavior. However other experiments show that wet stiffness is generally a more sensitive measure of fiber strength changes caused by hair treatments.

CONCLUSIONS

The Balanced Fiber method for measuring fiber stiffness offers simplicity in experimental setup, avoids need for fiber clamping and allows replicate measurements on single fibers. The measuring instrument may be as simple as a ruler or as complex as a traveling microscope. Fibers are not affected by stiffness measurements and can be measured for other physical properties or for changes caused by fiber treatments. The method appears readily adaptable for other materials in filament or sheet forms.

Hair fibers can be routinely compared for stiffness using only the distance measurement. However this parameter has theoretical significance which qualifies the method for use in research programs.

ACKNOWLEDGEMENT

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APPENDIX

LIST OF SYMBOLS

- A. Average cross-sectional area of fiber, cm^2
- D. Stiffness index, cm
- E_B. Elastic modulus for bending
- G. Stiffness coefficient
- H. Hookean slope for extension of a 5-cm fiber, g/m
- I. Moment of inertia of the fiber cross-sectional area
- L. Linear density of fiber, μg/cm

M. Bending moment
r. Average radius of a fiber, cm
R. Radius of curvature
T. Tension applied to each fiber end, dynes
W. Diameter of wire used as support, cm
Z. Fiber length, cm
e. Tensile strain on fiber
p. Bulk density of fiber, g/cm^3
σ. Tensile stress in fiber

FIBER SHAPE

Physics and engineering texts (24) commonly show equations derived for cantilever beams having very small beam deflections and such equations are often used as a basis for stiffness measurements on fibers. The restriction to small-beam deflections simplifies the derivation steps but, for the fiber hanging over a wire, infinite beam lengths and deflections must be considered in developing suitable theory.

Referring to Figure 1, the right half of the fiber suffices for derivation purposes and fiber weight is assumed negligible compared to the attached weight.

At any point on the fiber, the bending or clockwise moment, $T(D/2 - x)$, will oppose and at equilibrium will equal the restoring or counterclockwise moment, $E_B\,I/R$ where E_B is the elastic modulus for bending, I is the moment of inertia of the cross-sectional area and R is the radius of curvature of the bent fiber. Replacing R with the differential expression (24) for the radius of curvature of an arc, and collecting terms, we obtain

$$\frac{dp}{(1 + p^2)^{3/2}} = \frac{T}{E_B\,I}\left(\frac{D}{2} - x\right) dx \qquad (1)$$

where $p = dy/dx$. This is a standard form of differential which integrates to give

$$\frac{p}{(1 + p^2)^{1/2}} = \frac{T}{E_B\,I}\left(\frac{Dx}{2} - \frac{x^2}{2}\right) \qquad (2)$$

The integration constant is zero since at the hook x and p equal zero. At the weighted end of the fiber $x = D/2$, p is infinitely large and therefore from eq 2

$$\frac{T}{E_B\,I} = \frac{8}{D^2} \qquad (3)$$

Substituting $8/D^2$ for $T/E_B\,I$ in eq 2 and dy/dx for p gives

$$dy = \frac{\left(\frac{4x}{D} - \frac{4x^2}{D^2}\right) dx}{\left[1 - \left(\frac{4x}{D} - \frac{4x^2}{D^2}\right)^2\right]^{1/2}} \qquad (4)$$

An integrated expression is obtained from eq 4 by substituting $\cos\theta$ for $4x/D - 4x^2/D^2$ and integrating "by parts."

$$y = \frac{D}{\sqrt{2}} F - \frac{D}{4\sqrt{2}} \log_e \frac{1 + F}{1 - F} + C \qquad (5)$$

where

$$F = \left(\frac{1}{2} + \frac{2x}{D} - \frac{2x^2}{D^2}\right)^{1/2}$$

and the constant

$$C = \frac{-D}{2} + \frac{D}{4\sqrt{2}} \log_e \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

For us, the integration of eq 4 was an exercise requiring 40 steps and no effort was made to find a shorter route. Copies of the detailed integration are available for interested persons.

Equation 5 defines the shape taken by an ideal fiber when weighted at the ends and hung from a fine wire. Only distance D must be known to calculate y values at assigned values of x from zero to D/2.

When eq 5 is divided through by D, y/D is expressed as a function of x/D and y/D has a numerical value at each assigned x/D value. A computer print-out of these quantities is provided below. For any D measurements, x and y values along the fiber are readily obtained. Using negative x values, points are obtained for the left half of the fiber.

Factors for Computing Fiber Shape

x/D	−y/D
0.001	0.000002
0.002	0.000008
0.0025	0.000012
0.004	0.000032
0.005	0.000050
0.008	0.000127
0.010	0.000199
0.025	0.001232
0.050	0.004878
0.100	0.019335
0.200	0.079170
0.300	0.194416
0.400	0.423225
0.450	0.664291
0.475	0.908362
0.500	

FIBER LENGTH

The length z of fiber from the wire to any point along the fiber may also be expressed as a function of the D parameter. Using the differential expression for the length of an arc (25) and substituting the value for dy/dx from eq 4, we obtain

$$\frac{dz}{dx} = \left[1 - \left(\frac{4x}{D} - \frac{4x^2}{D^2}\right)^2\right]^{-1/2}$$

This expression may be integrated by trigonometric substitution to give

$$z = \frac{-D}{4\sqrt{2}} \log_e \frac{1+F}{1-F} + C' \tag{6}$$

where F is defined as for eq 5 and the integration constant

$$C' = \frac{D}{4\sqrt{2}} \log_e \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

ELASTIC MODULI

The elastic bending modulus E_B is expressed in eq 3 as a function of parameters, known experimentally except for I, the moment of inertia of the fiber cross section. In order to calculate E_B, the cross-sectional shape is assumed to be circular and hence $A^2/4\pi$ can be substituted for I to give

$$E_B = \frac{\pi T D^2}{2A^2} \tag{7}$$

The area A is estimated for each fiber by dividing linear density determinations in g/cm units by an assumed 1.31 g/cm^3 bulk density (11, 19).

The elastic stretch or tensile modulus E_S is calculated from

$$E_S = H\,g\,l/A\,\Delta \tag{8}$$

where the Hookean slope H is determined as described in the Experimental Section and the cross-sectional area A is estimated as for the bending modulus. The fiber length l is 5.0 cm, the fiber extension Δ is 0.1 cm and g is the gravitational constant.

STIFFNESS COEFFICIENT

The stiffness coefficient G may be preferred over the stiffness index for expression of results if different weights are needed to cover wide ranges of fiber stiffness. The index would require correction to a standard weight basis using the relation $D_1^2 = w_2 D_2^2/w_1$ where w_1 is the standard g weight and w_2 is the g weight used for the D measurement.

The coefficient is defined (26) as the ratio of an applied force to the displacement from equilibrium and equals E_B I. From eq 3 therefore

$$G = \frac{TD^2}{8} \tag{9}$$

where T is the applied force in dynes. The stiffness coefficient is more useful for practical comparisons than the bending modulus since the actual bending resistance of a fiber is represented without need to estimate fiber diameters and shapes.

FORCES ON THE FIBER

Bending stress and strain for a balanced fiber are greatest at the cross section above the wire where the radius of curvature R is a minimum. As shown earlier, at x = o, E_B

I/R = TD/2 and, from eq 3, EB I = $TD^2/8$. Combining the equations to eliminate E_B I, R = D/4. Strain e is equal to r/R, where r is the radius of the fiber, and hence e = 4r/D. Since the elastic modulus is the ratio of stress to strain, the maximum stress $\sigma = e\,E_B$. Substituting 4r/D for e and the value of E_B from eq 3 and since $I = \pi r^4/4$, we have for the maximum fiber stress

$$\sigma = \frac{2\,TD}{\pi r^3} \tag{10}$$

Theoretically and empirically, D is proportional to A or r^2 and consequently the maximum stress is inversely proportional to r. Thus, thin fibers undergo greater stresses than thick fibers and require more care in handling during measurements.

The stiffness index value and the maximum bending moment are affected by changes in the applied force T. Since $TD^2 = 8\,E_B\,I$ = a constant, a change from T to kT changes D to $D/\sqrt{k}$. The maximum bending moment M = TD/2. When T is changed to kT, D changes to $D/\sqrt{k}$ and, to maintain the equality 2M = TD, the maximum bending moment changes to $M\sqrt{k}$.

CONTACT OF FIBER WITH WIRE

Acceptable stiffness measurements require that contact between fiber and wire be minimal, theoretically a point contact. For a given bending force and fiber stiffness, this places a limit on wire diameter that may be used.

As shown above, the radius of curvature of the bent fiber R = D/4. For equal radius of curvature of wire and bent fiber, R must equal the wire radius or half its diameter. Replacing R with W/2 gives 2W = D. Accordingly, for "point" contact, wire diameter should be less than half the distance D. At larger wire sizes, contact between fiber and wire assumes an arc shape.

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