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Basic optics of effect materials

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Synopsis

Effect materials derive their color and effect primarily from thin-film interference. Effect materials have evolved over the decades from simple guanine crystals to the complex multilayer optical structures of today. The development of new complex effect materials requires an understanding of the optics of effect materials. Such an understanding would also benefit the cosmetic formulator as these new effect materials are introduced. The root of this understanding begins with basic optics. This paper covers the nature of light, interference of waves, thin-film interference, color from interference, and color travel.

INTRODUCTION

Classical color pigments produce color by absorbing select bands of the visible spectrum. The remaining wavelengths are diffusely reflected (scattered) and combine to produce a color. Many effect materials, on the other hand, derive their color from thin film interference. Examples of products that contain thin films include mica and borosilicate flakes coated with TiO_2 and/or Fe_2O_3, and more recently with multilayer stacks such as $TiO_2/SiO_2/TiO_2$. As a result of reflection and refraction of light on a thin film, certain wavelengths are removed by destructive interference while others are enhanced by constructive interference. This paper provides an understanding of how light interacts with a thin film to create interference and how color is derived from that interference.

NATURE OF LIGHT

LIGHT AS A WAVE

In physics class, we are taught about the dual nature of light, sometimes being considered a particle and at other times a wave. For the purposes of thin film interference, light is considered to be a wave as depicted in Figure 1. A wave has a wavelength λ, which is the length of a repeating unit (m) shown in Figure 1, a velocity v, which is the speed of the wave in a medium (m/s), and a frequency ν, which is the number of waves that pass per unit of time (s^{-1} or Hz). Wavelength, velocity, and frequency are related by equation 1:

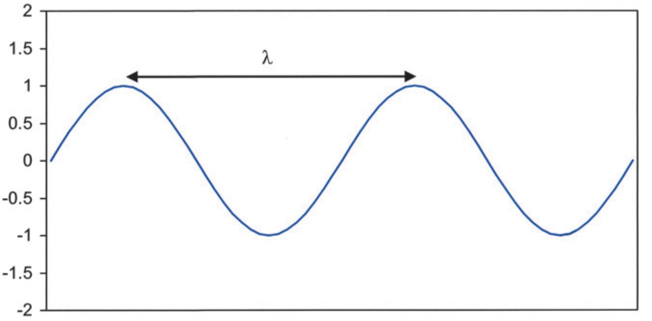

Figure 1. A light wave of wavelength λ.

$$v = \nu\lambda \tag{1}$$

It is important to note that the frequency is fixed by the light source and remains constant. Also note that wavelength and velocity are directly proportional and that wavelength and frequency are inversely proportional at constant velocity. For a wave traveling in a vacuum or in air (close enough to a vacuum), the velocity is c, and the speed of light, 3×10^8 m/s.

INDEX OF REFRACTION

The index of refraction is a unitless complex number ($n + ik$), where k is the absorption coefficient. For transparent, non-absorbing materials, $k = 0$ and the refractive index reduces to the simpler and more familiar n. The refractive index of a medium is defined as the speed of light in a vacuum divided by the velocity of light in the medium, as in equation 2:

$$n = c / v \tag{2}$$

The higher the index of refraction of a material, the slower the velocity of light through the material. The refractive index of a material is also dependent on the wavelength of the light. For white light, which contains many wavelengths, this phenomenon is referred to as dispersion. Therefore, tables of refractive indexes list values for one wavelength, commonly 589.3 nm, also referred to as the sodium D line.

REFLECTION AND REFRACTION

When light travels in a medium, it travels in a straight line until it hits an interface, defined as a change in the refractive index. At the interface, some of the light is reflected and some is refracted (see Figure 2).

The law of reflection states that the incident angle relative to the normal, θ_1, is equal to the reflected angle, θ_1'. This reflection is specular. If the reflection is at an interface where $n_2 > n_1$, the reflected wave has a phase shift of 180 degrees. The percentage of light

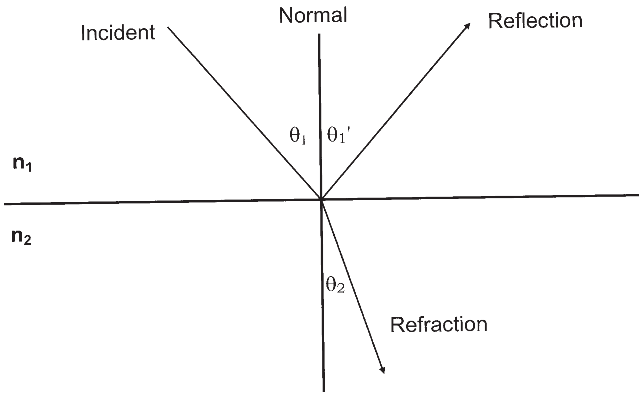

Figure 2. Reflection and refraction of incident light at the interface between media with refractive indexes of n_1 and n_2.

reflected at the interface is determined by the Fresnel equations. At normal incidence, the Fresnel equation simplifies to equation 3:

$$R = [(n_2 - n_1)/(n_2 + n_1)]^2 \tag{3}$$

The equation becomes more complicated for non-normal incidence. Regardless of the form, the point is the same: the larger the difference between n_1 and n_2, the higher the reflection will be. For example, at the interface between media with an index of refraction of 1 (air) and 2.7 (rutile titanium dioxide), the % reflection (at normal incidence) is 21%, while the % reflection (at normal incidence) at the interface between media with an index of refraction of 1 (air) and 1.5 (silica) is only 4%.

Light that is not reflected at the interface enters the medium and is refracted. The refraction angle θ_2 is determined by Snell's law in equation 4:

$$n_1 \mathrm{Sin}\theta_1 = n_2 \mathrm{Sin}\theta_2 \tag{4}$$

The velocity of light through a medium that is denser than space (or air) is slower than the maximum value for the speed of light. Equation 2 can be rearranged to solve for velocity as in equation 5:

$$v = c/n \tag{5}$$

This equation means that the higher the refractive index of a medium, the slower the velocity of light through that medium. If equation 5 is substituted into equation 1, the result is equation 6:

$$c/n = \nu\lambda \tag{6}$$

Since the frequency of a wave and the speed of light remain constant, the wavelength of light changes according to equation 7:

$$\lambda_2 = \lambda_1 (n_1/n_2) \tag{7}$$

INTERFERENCE OF WAVES

SINE WAVE

A light wave can be mathematically represented by the function in equation 8:

$$y = \sin(x) \tag{8}$$

One wavelength is depicted when the function is evaluated from 0 to 2π, or 360 degrees. Figure 3 shows the sine wave.

PHASE SHIFT

A phase shift is the number of degrees (radians) that a wave is shifted in the x direction. The sine wave function is modified, as in equation 9, where p is the phase shift:

$$y = \sin(x - p) \tag{9}$$

Because 2π radians and 360 degrees represent a full circle as well as a full wavelength, there are equivalent phase shifts such that

$$0 = 2\pi\,(360) = 4\pi\,(720) = 6\pi\,(1080);$$

$$\pi\,(180) = 3\pi\,(540) = 5\pi\,(900) = 7\pi\,(1260);\text{ and}$$

$$\pi/2\,(90) = 5\pi/2\,(450) = 9\pi/2\,(810) = 13\pi/2\,(1170)$$

The result of a phase shift on the original wave is depicted in the following figures. Figure 4 shows a phase shift of 45 degrees, Figure 5 shows a phase shift of 90 degrees, Figure 6 shows a phase shift of 135 degrees, and Figure 7 shows a phase shift of 180 degrees.

INTERFERENCE

Interference occurs when two waves are in same space at the same time. When two waves interfere, the resultant wave is the sum of the y values for each x value. It is very common

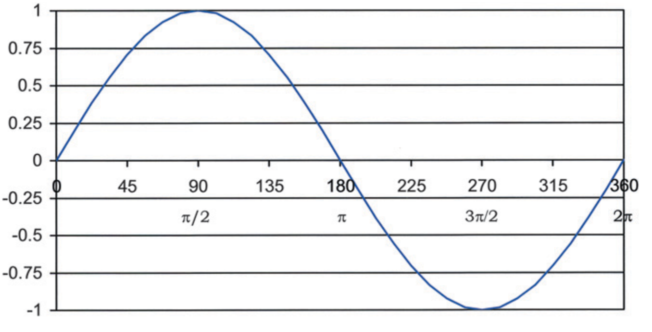

Figure 3. Depiction of a sine wave evaluated for one wavelength.

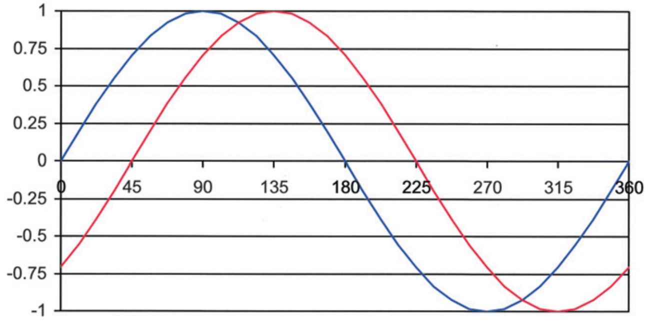

Figure 4. Original wave (blue line) compared to the wave shifted by 45 degrees (red line).

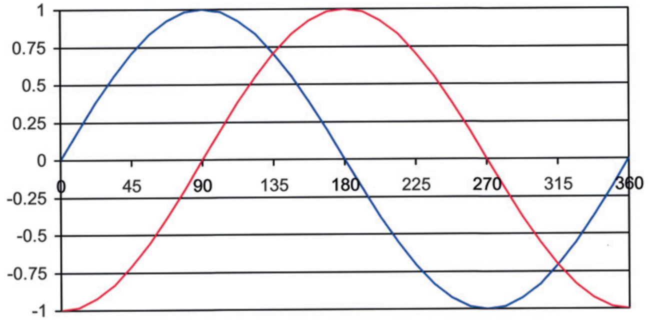

Figure 5. Original wave (blue line) compared to the wave shifted by 90 degrees (red line).

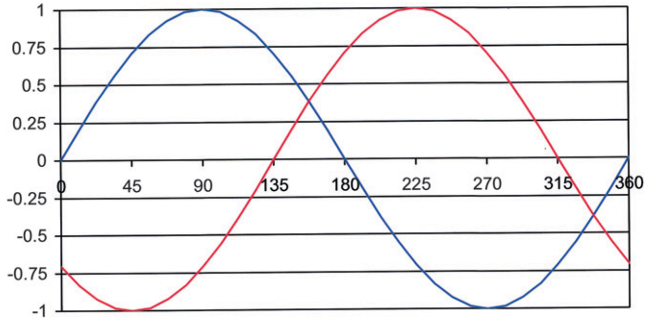

Figure 6. Original wave (blue line) compared to the wave shifted by 135 degrees (red line).

to have the phase of one wave shifted relative to the other. Figure 8 depicts a wave and a wave that is phase shifted by 0 degrees. The values of these two waves are added to give the wave that is the result of interference. It is important to note that the wavelength does not change as a result of interference, only the amplitude (maximum y value). As another example, Figure 9 depicts a wave and a wave that is phase shifted by 45 degrees. Again, the values of these two waves are added to give the wave that is the result of interference. Similarly, Figures 10, 11, and 12 show the result of interference in phases shifted by 90, 135, and 180 degrees, respectively. These figures are just a few examples of interference showing a range of amplitudes of the resulting waves. Among these, there are two special

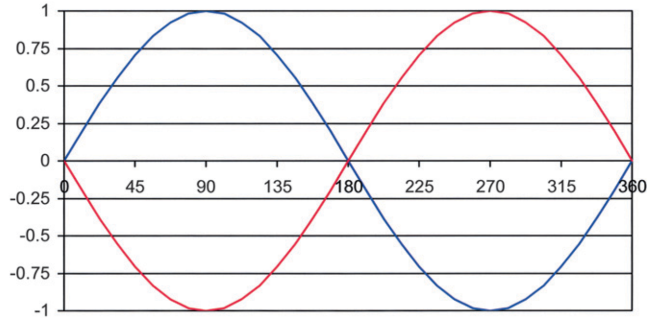

Figure 7. Original wave (blue line) compared to the wave shifted by 180 degrees (red line).

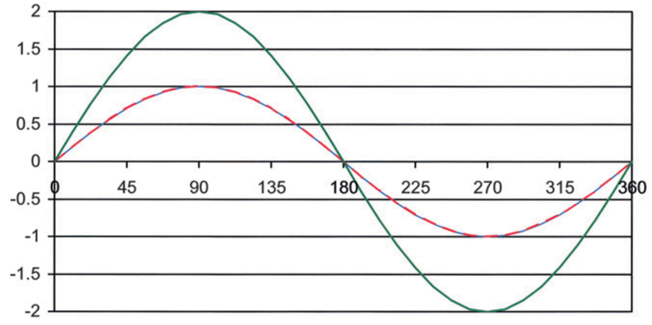

Figure 8. Original wave (blue line), the wave shifted by 0 degrees (red line), and the resultant wave of interference (green line).

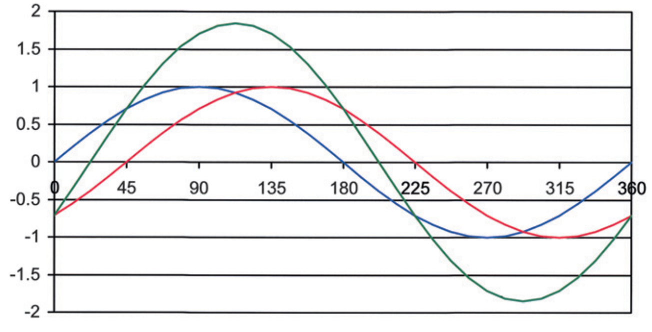

Figure 9. Original wave (blue line), the wave shifted by 45 degrees (red line), and the resultant wave of interference (green line).

cases. One, when the phase shift is 0 degrees, which results in the maximum amplitude of 2, is referred to as the condition of constructive interference. The other, when the phase shift is 180 degrees, which results in an amplitude of 0, is referred to as the condition of destructive interference. All other phase shifts result in partial interference, with amplitudes ranging between 0 and 2.

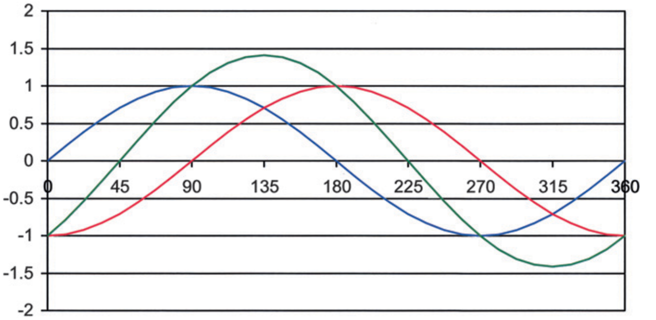

Figure 10. Original wave (blue line), the wave shifted by 90 degrees (red line), and the resultant wave of interference (green line).

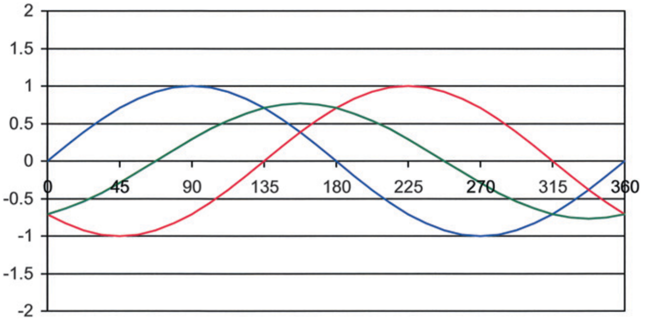

Figure 11. Original wave (blue line), the wave shifted by 135 degrees (red line), and the resultant wave of interference (green line).

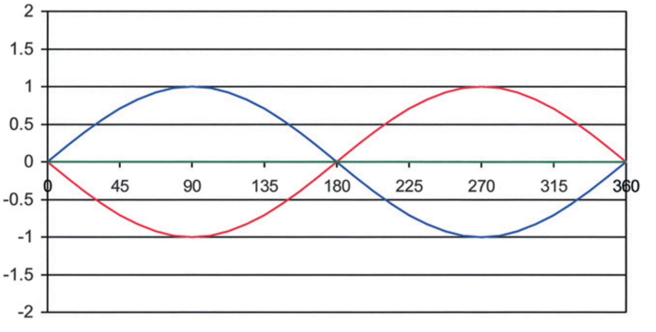

Figure 12. Original wave (blue line), the wave shifted by 180 degrees (red line), and the resultant wave of interference (green line).

THIN-FILM INTERFERENCE

GEOMETRY OF INTERFERENCE

There are many examples of thin-film interference in the natural world. Examples include soap bubbles and oil spilled on a wet road, as well as some birds and insects.

Figure 13 depicts a typical thin film, where $n_1 < n_2$. When light shines on the film, some of the light is reflected off the top surface and the rest of the light is refracted down into the film. When the light reaches the bottom interface of the film, some of the light is reflected back up. The rest of the light exits the film. When light waves that were reflected from the top meet the waves that traveled through the film, interference occurs.

PATHLENGTH AND PHASE SHIFT

The nature of the interference, i.e. constructive, destructive, or some degree of partial interference, is determined by the phase shift of the waves. The phase shift of the two interfering waves is determined by the pathlength difference, which means how much farther did the shifted wave travel to get to the point of interference compared to the other. With a first glance at Figure 13, one might assume that the pathlength difference is equal to the distance that W_{I_1} traveled in the film. However, this is not correct.

Figure 14 depicts the geometry of the pathlength difference in the thin film. The two incident waves, W_{I_1} and W_{I_2}, are parallel and in phase up to the point where W_{I_1} is at point

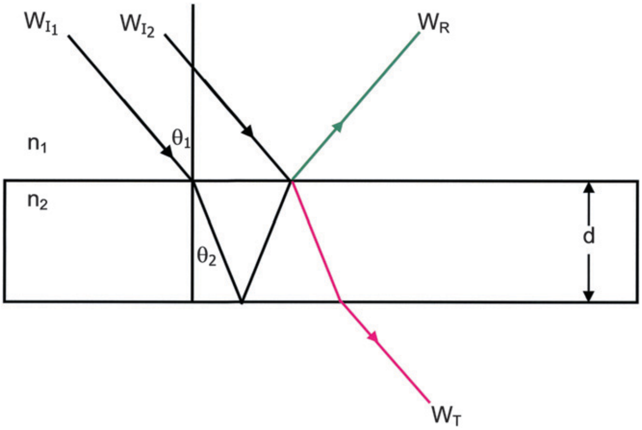

Figure 13. Thin-film interference.

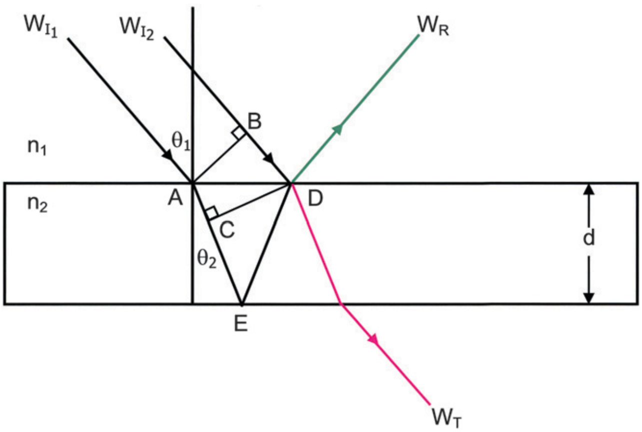

Figure 14. Geometry of pathlength difference in a thin film.

A and W_{I_2} is at point B. From this point, the two waves take different paths to reach point D, where interference occurs. While it is true that W_{I_1} travels the distance AE + ED to reach point D, W_{I_2} also travels the distance BD to reach point D. In the time it takes W_{I_2} to travel the distance BD, W_{I_1} travels the distance AC, which is determined by change in the wavelength in the film from equation 7; so the actual pathlength difference is determined by CE + ED. It turns out that the distance CE + ED can be conveniently indicated by equation 10:

$$\text{Pathlength difference} = 2d\text{Cos}\theta_2 \tag{10}$$

The phase shift is determined by converting the pathlength difference into a fraction of the wavelength in medium n_2 by multiplying by $2\pi/\lambda_2$, resulting in equation 11:

$$\text{Phase shift} = 4\pi d\text{Cos}\theta_2 / \lambda_2 \tag{11}$$

Note that the wavelength is in terms of λ_2, the effective wavelength while traveling in the film, because the pathlength difference occurs in the film. To make it more convenient, equation 7 can be substituted into equation 11, and assuming that $n_1 = 1$ for air, which will be the case going forward, the result is equation 12:

$$\text{Phase shift} = 4\pi n_2 d\text{Cos}\theta_2 / \lambda_1 \tag{12}$$

Now, the phase factor can be taken out of equation 12, leaving the final form of the pathlength difference in equation 13:

$$\text{Pathlength difference} = 2n_2 d\text{Cos}\theta_2 \tag{13}$$

Thus the pathlength difference and phase shift depend on the refractive index and thickness of the film (the product of these is known as optical thickness) and the refraction angle, which is linked back to the incident angle through equation 4.

REFLECTION INTERFERENCE

Reflection interference (W_R) results at point D in Figure 14 when the reflected portion of W_{I_2} meets the externally refracted portion of W_{I_1}. These two waves will constructively interfere if they are in phase. Since W_{I_2} reflects off an interface where $n_2 > n_1$, it has a phase shift of 180 degrees, which is equivalent to a pathlength difference of $^1/_2\lambda$. Therefore, the net pathlength difference for W_{I_1} must also be $^1/_2\lambda$. Thus the equation for constructive interference is in equation 14:

$$(m + \tfrac{1}{2})\lambda_1 = 2n_2 d\cos\theta_2 \quad (m = 0,1,2,3\ldots) \tag{14}$$

Destructive interference of these two waves will occur if they are out of phase by 180 degrees. Again, W_{I_2} reflects with a phase shift of 180 degrees, and therefore, the net pathlength difference for W_{I_1} must be λ. Consequently the equation for destructive interference is in equation 15:

$$m\lambda_1 = 2n_2 d\cos\theta_2 \quad (m = 1,2,3,4\ldots) \tag{15}$$

The following example demonstrates how to determine the wavelengths that in reflection are either constructively or destructively interfered for light that is normal incident from air on a thin film with a refractive index of 2.5 and a thickness of 255 nm.

Solution. For normal incidence, $\theta_2 = 0$. The pathlength from equation 13 is then $2 \times 2.5 \times 255 \times \mathrm{Cos}(0) = 1275$ nm.

Use constructive interference equation 14 and plug in various values of m and solve for λ_1:

$$(\tfrac{1}{2})\lambda_1 = 1275; \lambda_1 = 2550 \text{ nm}$$
$$(1\tfrac{1}{2})\lambda_1 = 1275; \lambda_1 = 850 \text{ nm}$$
$$(2\tfrac{1}{2})\lambda_1 = 1275; \lambda_1 = 510 \text{ nm}$$
$$(3\tfrac{1}{2})\lambda_1 = 1275; \lambda_1 = 364.29 \text{ nm}$$

Use destructive interference equation 15 and plug in various values of m and solve for λ_1:

$$(1)\lambda_1 = 1275; \lambda_1 = 1275 \text{ nm}$$
$$(2)\lambda_1 = 1275; \lambda_1 = 637.5 \text{ nm}$$
$$(3)\lambda_1 = 1275; \lambda_1 = 425 \text{ nm}$$
$$(4)\lambda_1 = 1275; \lambda_1 = 318.75 \text{ nm}$$

TRANSMISSION INTERFERENCE

Transmission interference (W_T) also results at point D in Figure 14 when the refracted portion of W_{I_2} meets the internally reflected portion of W_{I_1}. These two waves will constructively interfere if they are in phase. Since neither wave has reflected off an interface where $n_2 > n_1$, neither wave is phase shifted. Therefore, the net pathlength difference for W_{I_1} must be λ. Thus the equation for constructive interference is in equation 16:

$$m\lambda_1 = 2n_2 d\cos\theta_2 \quad (m = 1,2,3,4\ldots) \tag{16}$$

Destructive interference of these two waves will occur if they are out of phase by 180 degrees. Again, neither wave has a phase shift of 180 degrees; therefore, the net pathlength difference for W_{I_1} must be $^1/_2\lambda$. So then, the equation for destructive interference is in equation 17:

$$(m + \tfrac{1}{2})\lambda_1 = 2n_2 d\cos\theta_2 \quad (m = 0,1,2,3\ldots) \tag{17}$$

The following example demonstrates how to determine the wavelengths that in transmission are either constructively or destructively interfered for light that is normal incident from air on a thin film with a refractive index of 2.5 and a thickness of 255 nm.

Solution. For normal incidence, $\theta_2 = 0$. The pathlength from equation 13 is then $2 \times 2.5 \times 255 \times \mathrm{Cos}(0) = 1275$ nm.

Use constructive interference equation 16 and plug in various values of m and solve for λ_1:

$$(1)\lambda_1 = 1275; \lambda_1 = 1275 \text{ nm}$$
$$(2)\lambda_1 = 1275; \lambda_1 = 637.5 \text{ nm}$$
$$(3)\lambda_1 = 1275; \lambda_1 = 425 \text{ nm}$$
$$(4)\lambda_1 = 1275; \lambda_1 = 318.75 \text{ nm}$$

Use destructive interference equation 17 and plug in various values of m and solve for λ_1:

$$(\tfrac{1}{2})\lambda_1 = 1275; \lambda_1 = 2550 \text{ nm}$$
$$(1\tfrac{1}{2})\lambda_1 = 1275; \lambda_1 = 850 \text{ nm}$$
$$(2\tfrac{1}{2})\lambda_1 = 1275; \lambda_1 = 510 \text{ nm}$$
$$(3\tfrac{1}{2})\lambda_1 = 1275; \lambda_1 = 364.29 \text{ nm}$$

CONSERVATION OF ENERGY

One may have noticed that the solutions to the preceding examples for reflection and transmission have an inverse relationship. The condition for constructive interference in reflection is the same as for destructive interference in transmission, and the condition for destructive interference in reflection is the same as for constructive interference in transmission. Energy is neither created nor destroyed by interference, it is just redistributed. Two waves interfere and two waves result, as in equation 18:

$$W_{I_1} + W_{I_2} = W_R + W_T \tag{18}$$

In other words, for cases where all layers are non-absorbing, then what is not reflected is transmitted. This inverse relationship between reflection and transmission gives rise to complementary colors.

COLOR FROM INTERFERENCE

White light contains a continuum of wavelengths from 400 to 700 nm. For simplicity, white light can be broken down into the three primary colors and assigned an average wavelength: blue = 450 nm, green = 550 nm, and red = 650 nm. In Figure 15, the intensity of each of the three primary colors is plotted versus pathlength difference. The maxima and minima for each primary color correlate to the conditions for constructive and destructive interference, respectively. Conditions for constructive interference occur when the pathlength difference equals 0.5λ and 1.5λ, etc., and destructive interference occurs when the pathlength difference equals λ and 2λ, etc. For example, look at the curve for the blue primary. Here, 0.5λ and 1.5λ are 225 and 675 nm for constructive interference, and λ and 2λ are 450 and 900 nm for destructive interference. All other points on the curve represent some degree of partial interference.

Note about the intensity values in Figure 15: The original wave in Figure 3 has an amplitude of one. Upon constructive interference, the resulting wave has an amplitude of two (see Figure 8). The intensity is the square of the amplitude, thus resulting in four. Also note that Figure 15 applies to reflection interference only.

Of all the wavelengths in white light, only a few of the wavelengths are actually constructively or destructively interfered for a particular film, with the rest being partially interfered. The observed interference color of a particular film is then the combined effect of adding the partial interferences plus the few constructive and destructive ones. The simplified primary colors in Figure 15 also demonstrate this principle.

Figure 16 highlights particular cases of pathlength differences. There are infinite combinations of refractive index, film thickness, and incident angle that can produce a given pathlength difference. For simplicity, the refractive index of the film will be fixed at 2.5,

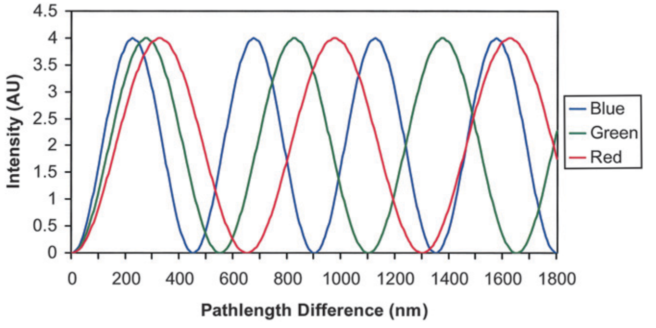

Figure 15. Intensity of the three primary colors versus pathlength difference.

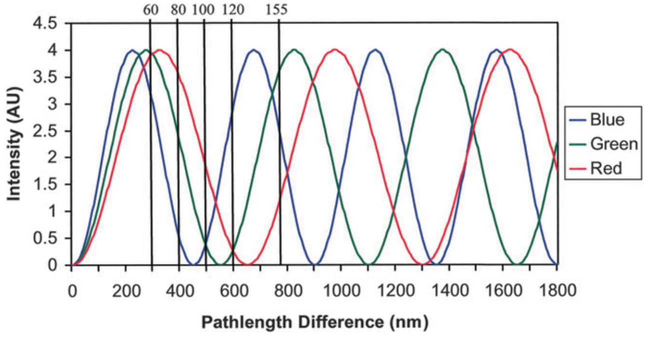

Figure 16. Highlighted pathlengths for selected film physical thicknesses. The thickness of each film is indicated on top of the highlighted line.

the incident angle will be held at normal, and only the film thickness will vary. This simplification is in fact the reality of effect material production. The refractive index is dictated by the material of choice and samples are always measured at a fixed angle. As a film is grown, only the thickness changes. Figure 16 highlights film thicknesses of 60, 80, 100, 120, and 155 nm.

If the relative intensities of the primary colors for each are plotted, one can see how the resulting color caused by interference is obtained. The primary color intensities for the 60-nm film are plotted in Figure 17. High intensities of red, green, and blue add to white interference, also referred to as pearl. The intensities of other wavelengths are added to fill out the spectral curve. The primary color intensities for the 80-nm film are plotted in Figure 18. High intensities of red, green, and low-intensity blue add to yellow interference, also referred to as gold. The intensities of other wavelengths are added to fill out the spectral curve. The primary color intensities for the 100-nm film are plotted in Figure 19. A high intensity of red and low intensities of green and blue add to red interference. The intensities of other wavelengths are added to fill out the spectral curve. There is a blue component observed of less than 450 nm that makes this color technically magenta. The primary color intensities for the 120-nm film are plotted in Figure 20. A high intensity of blue and low intensities of red and green add to blue interference. The intensities of other wavelengths are added to fill out the spectral curve. The primary color intensities for the 155-nm film are plotted in Figure 21. A high intensity of green and low intensities of red and blue add to green interference. The intensities of other wavelengths are added to fill out the spectral curve.

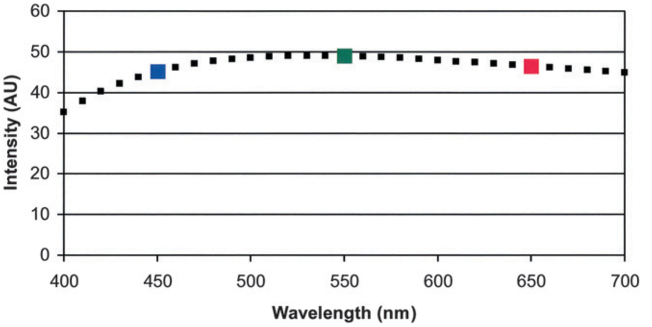

Figure 17. Primary color intensities (blue, green, red) for the 60-nm film, plus other wavelengths to fill out the pearl spectral curve.

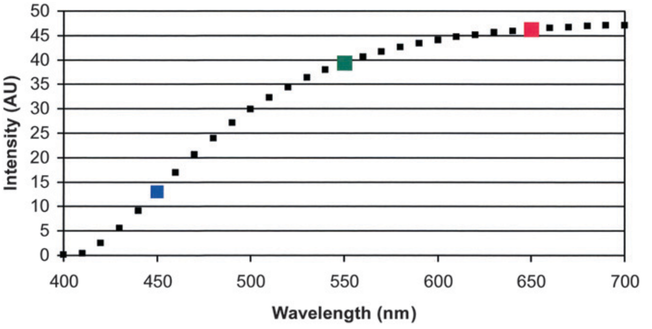

Figure 18. Primary color intensities (blue, green, red) for the 80-nm film, plus other wavelengths to fill out the gold spectral curve.

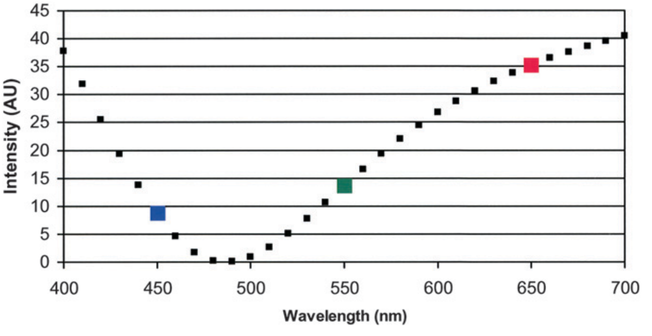

Figure 19. Primary color intensities (blue, green, red) for the 100-nm film, plus other wavelengths to fill out the red spectral curve.

The color series depicted in the preceding figures represents the nature of an effect material production. As the film is made thicker, the interference color changes accordingly. Figure 22 shows this color progression in more detail. Here, the color is plotted starting at 60 nm and progressing every 5 nm.

Spectral curves can also be calculated. The following example demonstrates how to calculate the reflectance spectral curve at normal incidence for a thin film (in air) with a refractive index of 2.5 and a thickness of 255 nm.

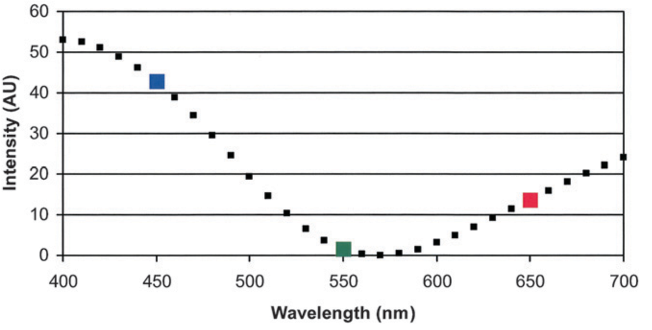

Figure 20. Primary color intensities (blue, green, red) for the 120-nm film, plus other wavelengths to fill out the blue spectral curve.

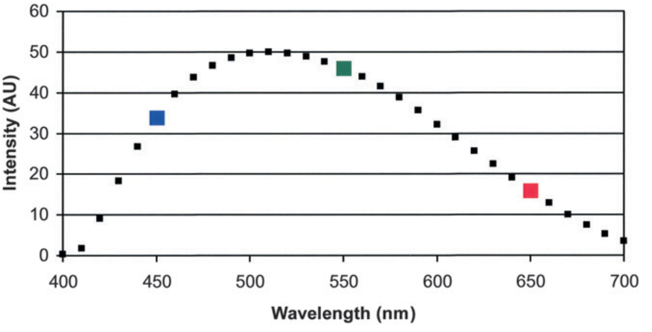

Figure 21. Primary color intensities (blue, green, red) for the 155-nm film, plus other wavelengths to fill out the green spectral curve.

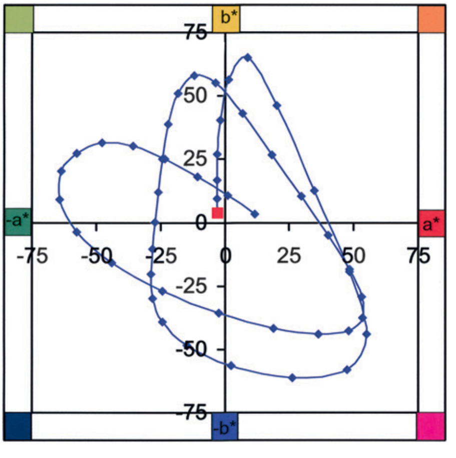

Figure 22. Interference color progression for a film starting at 60 nm (red square) and stepping every 5 nm.

The first step is to use equation 13 to determine the pathlength difference of the film: $2 \times 2.5 \times 255 \times \mathrm{Cos}(0) = 1275$ nm.

Next, the intensities of various wavelengths need to be determined and plotted. For this example, the intensities for the primary colors, 450 nm, 550 nm, and 650 nm are worked out.

Starting with 450 nm, first determine the phase shift by taking $2\pi/\lambda \times$ pathlength and adding π for constructive interference:

$$(2\pi/450 \times 1275) + \pi = 20.9440 \text{ radians}$$

$$\text{or, } 360/2\pi \times 20.9440 = 1200 \text{ degrees}$$

Next the result of interference is calculated and plotted. Table I contains the calculation and Figure 23 shows the plot for 450 nm. The amplitude of the resulting wave is the maximum magnitude, which in this example is at 150 degrees. Since the value at 150 degrees was not previously calculated, it is calculated here:

$$\text{Amplitude} = \text{Sin}(150) + \text{Sin}(150 - 1200) = 1.00$$

The intensity, which is the square of the amplitude, equals $(1.00)^2 = 1.00$.

Similarly, with 550 nm, first determine the phase shift by taking $2\pi/\lambda \times$ pathlength and adding π for constructive interference:

$$(2\pi/550 \times 1275) + \pi = 17.7072 \text{ radians}$$

$$\text{or, } 360/2\pi \times 17.7072 = 1015 \text{ degrees}$$

Again, the result of interference is calculated and plotted. Table II contains the calculation and Figure 24 shows the plot for 550 nm. The amplitude of the resulting wave is the

Table I
Calcuation for 450 nm

x	y = sin(x)	y = sin(x-1200)	Sum
0	0	−0.86603	−0.86603
45	0.7071	−0.96593	−0.25882
90	1	−0.5	0.5
135	0.7071	0.258819	0.965926
180	0	0.866025	0.866025
225	−0.7071	0.965926	0.258819
270	−1	0.5	−0.5
315	−0.7071	−0.25882	−0.96593
360	0	−0.86603	−0.86603

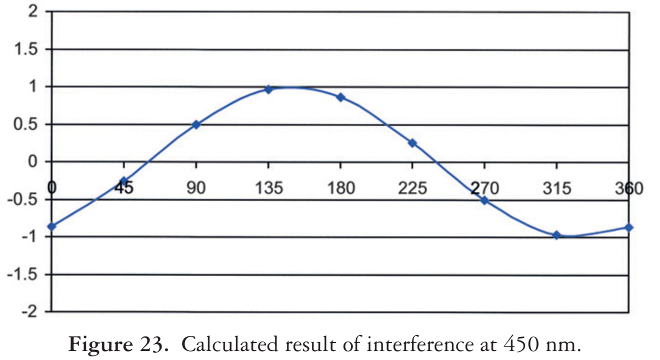

Figure 23. Calculated result of interference at 450 nm.

Table II
Calcuation for 550 nm

x	y = sin(x)	y = sin(x-1015)	Sum
0	0	0.906308	0.906308
45	0.7071	0.939693	1.646799
90	1	0.422618	1.422618
135	0.7071	−0.34202	0.365087
180	0	−0.90631	−0.90631
225	−0.7071	−0.93969	−1.6468
270	−1	−0.42262	−1.42262
315	−0.7071	0.34202	−0.36509
360	0	0.906308	0.906308

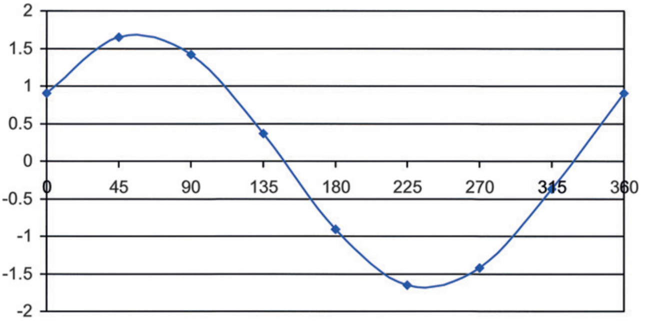

Figure 24. Calculated result of interference at 550 nm.

maximum magnitude, which in this example is at 60 degrees. Since the value at 60 degrees was not previously calculated, it is calculated here:

$$\text{Amplitude} = \text{Sin}(60) + \text{Sin}(60 - 1015) = 1.69$$

The intensity, which is the square of the amplitude, equals $(1.69)^2 = 2.86$.

With 650 nm, determine the phase shift by taking $2\pi/\lambda \times$ pathlength and adding π for constructive interference:

$$(2\pi / 650 \times 1275) + \pi = 15.4663 \text{ radians}$$

$$\text{or, } 360 / 2\pi \times 15.4663 = 886 \text{ degrees}$$

Finally, the result of interference is calculated and plotted. Table III contains the calculation and Figure 25 shows the plot for 650 nm. The amplitude of the resulting wave is the maximum magnitude, which in this example is at 180 degrees. Since the value at 180 degrees was previously calculated, it is read from Table III. The intensity, which is the square of the amplitude, equals $(0.242)^2 = 0.06$.

The intensities for the three primary colors are plotted to determine the total color of the interference film. In addition, one can add the respective intensities for the wavelengths that correspond to constructive and destructive interference. These wavelengths were determined in the example in the section on reflective interference. The constructive

Table III
Calcuation for 650 nm

x	y = sin(x)	y = sin(x-886)	Sum
0	0	−0.24192	−0.24192
45	0.7071	−0.85717	−0.15006
90	1	−0.9703	0.029704
135	0.7071	−0.51504	0.192069
180	0	0.241922	0.241922
225	−0.7071	0.857167	0.150061
270	−1	0.970296	-0.0297
315	−0.7071	0.515038	-0.19207
360	0	−0.24192	-0.24192

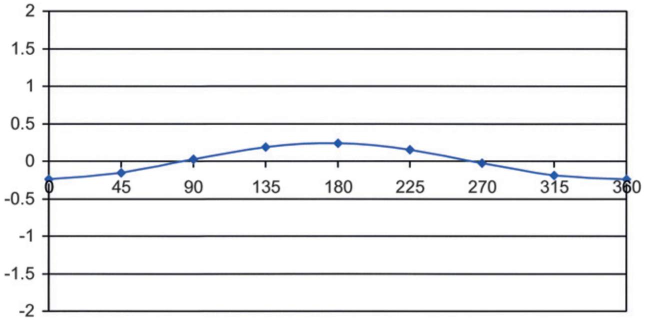

Figure 25. Calculated result of interference at 650 nm.

interference is at 510 nm, with an intensity of four. The destructive interference is at 425 nm and 637.5 nm, with an intensity of zero. These values are plotted in Figure 26, where the intensities of other wavelengths were added to fill in the curve.

COLOR TRAVEL

As discussed earlier, the pathlength difference of equation 13 (and therefore the color) depends on the refractive index and thickness of the film and the refraction angle, which can be tied back to incident angle. Once an effect material is made, its thickness and refractive index remain constant, but the incident angle can change, and therefore the color will change accordingly. Effect materials based on thin-film interference are said to have color travel because the color moves with changing incident angle. The degree of color travel depends on the refractive index. High-refractive-index materials have short travel and low-refractive-index materials have longer travel. When the incident angle is varied from normal incident toward a grazing angle, the pathlength gets shorter and the color travels backwards, which is opposite to the progression in Figure 22. Table IV contains a comparison of a low-refractive-index film with a high-refractive-index film with the same optical thickness. As the incident angle is changed, the refraction angle is calculated using equation 4, followed by the pathlength.

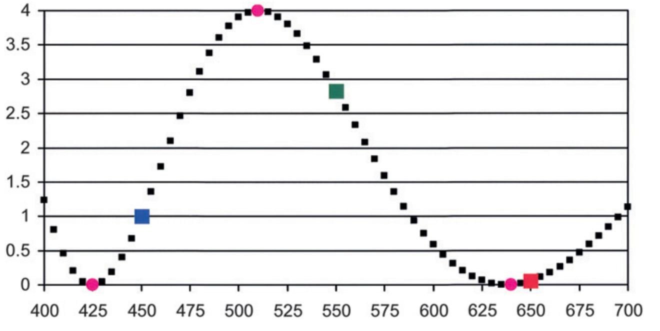

Figure 26. Resulting calculated spectral curve for 255-nm-thick interference film with a refractive index of 2.5. Also shown are the intensities of the primary colors (blue, green, red) and the constructive and destructive interference cases (●). The intensities of other wavelengths are used to fill out the curve.

Table IV
Pathlength vs Incident Angle

	SiO_2, n = 1.5, d = 250 nm		TiO_2, n = 2.5, d = 150 nm	
θ_1	θ_2	2nd Cos θ_2	θ_2	2nd Cos θ_2
0	0.0	750	0.0	750
10	6.6	745	4.0	748
20	13.2	730	7.9	743
30	19.5	707	11.5	735
40	25.4	678	14.9	725
50	30.7	645	17.8	714
60	35.3	612	20.3	704
70	38.8	585	22.1	695
80	41.0	566	23.3	689

Comparing the data in Table IV reveals that the refraction angle for SiO_2 is greater than for TiO_2 at any incident angle except for normal, where they are equal. This difference is a direct consequence to Snell's Law of equation 4. With the optical thickness of SiO_2 and TiO_2 set to be equal, they have the same pathlength (and color) at normal incident. At other angles, the differences in refraction angle lead to different pathlengths (and colors). The resulting pathlengths for the low-refractive SiO_2 have a larger range than for the higher-refractive TiO_2, and thus the SiO_2 has more color travel. The differences in color travel are better demonstrated on a color chart, as in Figure 27. While both TiO_2 and SiO_2 start out with the same green color, TiO_2 only travels to about a blue while SiO_2 travels all the way to orange.

Another way to demonstrate color travel is to look at the spectral curves. Figure 28 contains the spectral curves for a TiO_2 film with the same optical thickness as in the example from the section "Color From Interference," so that the curve at normal is the same as that in Figure 26. This curve is the one farthest to the right in Figure 28. The primary colors are highlighted. Each curve to the left is a result of a 10-degree incident angle change from the curve to the right. Note that the overall shape of the curve is not changing; it is just moving to the left and the intensities of the primary colors go up and down accordingly, like buoys in the ocean as waves go by. Initially the curve has high intensity for green and low intensity for blue and red, resulting in a green color. As the curve shifts,

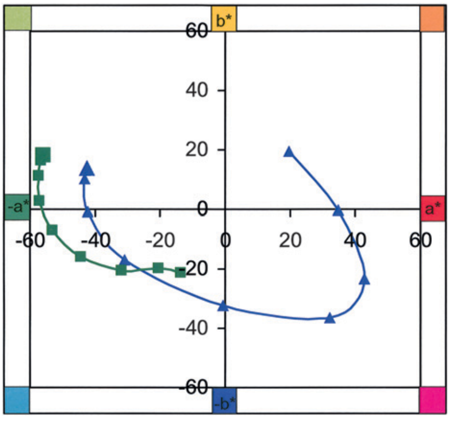

Figure 27. Color travel plot for TiO_2 (■) and SiO_2 (▲). The normal angles are indicated with larger markers.

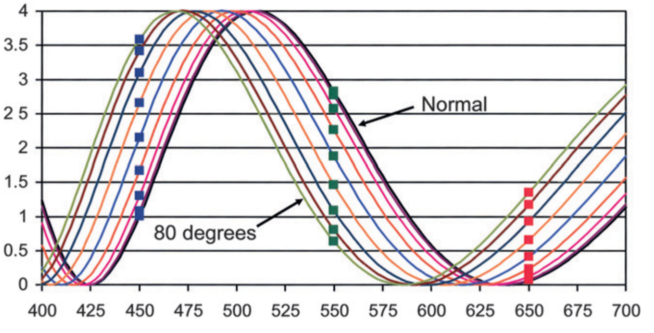

Figure 28. Shifting color curves due to incident angle for a TiO_2 film. The dark blue curve farthest to the right is at normal and the olive green curve farthest to the left is for 80 degrees. Others are at 10-degree steps. Also shown are the intensities of the primary colors (blue, green, red).

the intensity of green decreases and the intensity of blue increases. The red, while increasing, remains low. The result of the shift is color travel.

For comparison, Figure 29 contains the spectral curves for a SiO_2 film with the same optical thickness as the TiO_2 film above. Again, the curve at normal is the one farthest to the right in Figure 29, and the primary colors are highlighted. Initially the curve has high intensity for green and low intensity for blue and red, resulting in a green color. As the curve begins to shift, the intensity of green decreases and the intensity of blue increases. At higher angles, the intensity of green hits a minimum and then increases while the intensity of blue hits a maximum and then decreases. Red, meanwhile, continually increases from the initial low value to a maximum. The result of the shift is color travel that is captured in Table V. Compare the curves in Figure 28 to those in Figure 29. Those in Figure 29 with the low-refractive-index SiO_2 move much farther than those in Figure 28 with the high-refractive-index TiO_2.

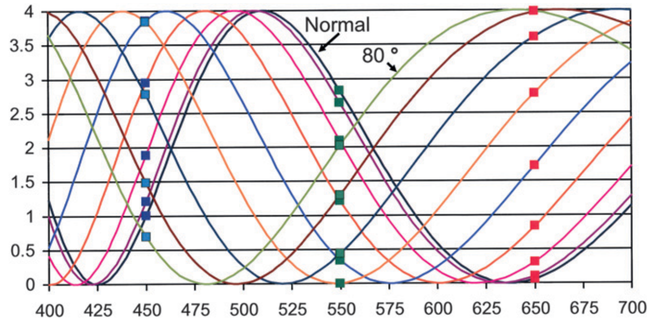

Figure 29. Shifting color curves due to incident angle for a SiO_2 film. The dark blue curve farthest to the right is at normal and the olive green curve farthest to the left is for 80 degrees. Others are at 10-degree steps. Also shown are the intensities of the primary colors (blue, green, red). The light green primary highlights with the black square are increasing after minimum. The light blue primary highlights with the black square are decreasing after maximum.

Table V
Resulting Color Due to Shifting Incident Angles for SiO_2 Film

Angle	Blue	Green	Red	Result
0	Low	High	Low	Green
10	Low	High	Low	Green
20	Medium	Medium	Low	Blue-green
30	High	Medium	Low	Green-blue
40	High	Low	Medium	Red-blue
50	High	Low	High	Violet
60	Medium	Low	High	Magenta
70	Medium	Medium	High	Red
80	Low	Medium	High	Orange

SUMMARY

Effect materials derive their color and effect primarily from thin-film interference. Interference occurs when an effect material has at least one thin optically active layer. When light impinges on the layers of an effect material, light is both reflected and refracted. How the light is reflected and refracted depends on the refractive indexes of the layer and of the medium. The interference of the wave is dictated by the pathlength difference, $2n_2d\mathrm{Cos}(\theta_2)$. Depending of the pathlength difference, a given wave can be constructively interfered, destructively interfered, or more likely, to some degree partially interfered. The color from interference comes from adding the colors from the resulting interference from each wave across the spectrum. Effect materials based on thin-film interference will have a reflection interference color and a transmission interference color, which are complementary to each other. Since the color is dictated by the pathlength difference, color depends on the refraction angle, which corresponds to the incident angle. Therefore, color is angle-dependent and is said to travel as the incident angle is varied.

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